

GLOBAL WELL-POSEDNESS ON THE DERIVATIVE NONLINEAR SCHRÖDINGER EQUATION

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ABSTRACT. As a continuation of the previous work [18], we consider the global well-posedness for the derivative nonlinear Schrödinger equation. We prove that it is globally well-posed in energy space, provided that the initial data $u_0 \in H^1(\mathbb{R})$ with $\|u_0\|_{L^2} < 2\sqrt{\pi}$.

1. INTRODUCTION

We study the following Cauchy problem of the nonlinear Schrödinger equation with derivative (DNLS):

$$\begin{cases} i\partial_t u + \partial_x^2 u = i\partial_x(|u|^2 u), & t \in \mathbb{R}, x \in \mathbb{R}, \\ u(0, x) = u_0(x) \in H^1(\mathbb{R}). \end{cases} \quad (1.1)$$

It arises from studying the propagation of circularly polarized Alfvén waves in magnetized plasma with a constant magnetic field, see [11, 12, 14] and the references therein. The problem (1.1) is L^2 -critical, and the equation is complete integrable. Moreover, the H^1 -solution of (1.1) obeys the following mass, energy, and momentum conservation laws,

$$M_D(u(t)) := \int_{\mathbb{R}} |u(t)|^2 dx = M_D(u_0), \quad (1.2)$$

$$E_D(u(t)) := \int_{\mathbb{R}} \left(|u_x(t)|^2 + \frac{3}{2} \operatorname{Im} |u(t)|^2 u(t) \overline{u_x(t)} + \frac{1}{2} |u(t)|^6 \right) dx = E_D(u_0), \quad (1.3)$$

$$P_D(u(t)) := \operatorname{Im} \int_{\mathbb{R}} \bar{u}(t) u_x(t) dx - \frac{1}{2} \int_{\mathbb{R}} |u(t)|^4 dx = P_D(u_0). \quad (1.4)$$

Local well-posedness for the Cauchy problem (1.1) is well-understood. It was proved in the energy space $H^1(\mathbb{R})$ by Hayashi and Ozawa in [7, 8, 9], see also Guo and Tan [6] for earlier result in smooth spaces. The problem for rough data below the energy space, see [3, 15, 16] for local well-posedness and ill-posedness.

The global well-posedness for (1.1) has been also widely studied. By using mass and energy conservation laws, and by developing the gauge transformations, Hayashi

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and Ozawa [9, 13] proved that the problem (1.1) is globally well-posed in energy space $H^1(\mathbb{R})$ under the condition

$$\|u_0\|_{L^2} < \sqrt{2\pi}. \quad (1.5)$$

Here 2π is the mass of the ground state Q of the corresponding elliptic problem. That is, Q is the unique (up to some symmetries) positive solution of the following elliptic equation

$$-Q_{xx} + Q - \frac{3}{16}Q^5 = 0. \quad (1.6)$$

Since $\sqrt{2\pi}$ is just the mass of the ground state, the condition of (1.5) was naturally used before to keep the energy to be positive.

However, the condition (1.5) was improved recently in our previous work [18]. We prove that there exists a small constant $\varepsilon_* > 0$, such that the problem (1.1) is still globally well-posed in energy space when the initial data satisfies $\|u_0\|_{L^2} < \sqrt{2\pi} + \varepsilon_*$. The result implies that for the problem (1.1), the ground state mass 2π is not the threshold of the global well-posedness and blow-up. This is totally different from the L^2 -critical power-type Schrödinger equation (the nonlinearity $i\partial_x(|u|^2u)$ in (1.1) replaced by $-\frac{3}{16}|u|^4u$), see [18] for some further discussion. Results on the global well-posedness when the initial data of regularity below the energy space, see [4, 5, 10, 16] for examples.

In this paper, we continue to consider the L^2 -assumption on initial data and obtain the global well-posedness as follows.

Theorem 1.1. *For any $u_0 \in H^1(\mathbb{R})$ with*

$$\int_{\mathbb{R}} |u_0(x)|^2 dx < 4\pi, \quad (1.7)$$

the Cauchy problem (1.1) is globally well-posed in $H^1(\mathbb{R})$ and the solution u satisfies

$$\|u\|_{L_t^\infty H_x^1} \leq C(\|u_0\|_{H^1}).$$

Since $4\pi = 2\|Q\|_{L^2}^2$, we in fact prove that the Cauchy problem (1.1) is globally well-posed in $H^1(\mathbb{R})$, when $\int_{\mathbb{R}} |u_0(x)|^2 dx < 2 \int_{\mathbb{R}} Q(x)^2 dx$.

Developing by Hayashi and Ozawa, the gauge transformation is one of element tools to study the derivative nonlinear Schrödinger equation. Let

$$v(t, x) := e^{-\frac{3}{4}i \int_{-\infty}^x |u(t, y)|^2 dy} u(t, x), \quad (1.8)$$

then from (1.1), v is the solution of

$$i\partial_t v + \partial_x^2 v = \frac{i}{2}|v|^2 v_x - \frac{i}{2}v^2 \bar{v}_x - \frac{3}{16}|v|^4 v, \quad (1.9)$$

with the initial data $v_0 = e^{-\frac{3}{4}i \int_{-\infty}^x |u_0|^2 dy} u_0$. Moreover, v obeys the same mass conservation law (1.2), while the energy conservation law (1.3) is deduced to

$$E(v(t)) := \|v_x(t)\|_{L_x^2}^2 - \frac{1}{16}\|v(t)\|_{L_x^6}^6 = E(v_0), \quad (1.10)$$

and the momentum conservation law (1.4) is deduced to

$$P(v(t)) := \operatorname{Im} \int_{\mathbb{R}} \bar{v}(t) v_x(t) dx + \frac{1}{4} \int_{\mathbb{R}} |v(t)|^4 dx = P(v_0). \quad (1.11)$$

From the argument used in [18], to prove the global well-posedness for the DNLS, an important ingredient is the usage of the momentum conservation law. We observe that the key point is to give a small control of the following term from (1.11),

$$\operatorname{Im} \int_{\mathbb{R}} \bar{v}(t) v_x(t) dx. \quad (1.12)$$

Or, to be more exact, one may prove that

$$-\operatorname{Im} \int_{\mathbb{R}} \bar{v}(t) v_x(t) dx \leq c \|v_x(t)\|_{L^2} \|v(t)\|_{L^2}, \quad (1.13)$$

where c is positive constant. It is trivial when $c = 1$ by Hölder's inequality. Suppose that one can get the bound with a suitable small constant c , then the global well-posedness were followed. In [18], by using a variational argument, we in fact proved that if the mass is larger but close to 2π , and there is a time sequence $\{t_n\}$ such that $\|v(t_n)\|_{H^1}$ tends to infinity, then $v(t_n)$ is close to Q up to some symmetries. Since Q is real-valued, (1.13) can be given for small $c > 0$.

In this paper, we give a different argument to prove the bound (1.13), under some suitable but explicit assumption on L^2 -norm of the initial data. Our method here do not need to use the property of the ground state Q of (1.6). To give an explanation on our argument, we say that if $\|v(t)\|_{H^1}$ tends to infinity, then by the momentum and energy conservation laws, (1.13) is roughly deduced to

$$\frac{1}{4} \|v(t)\|_{L^4}^4 \approx -\operatorname{Im} \int_{\mathbb{R}} \bar{v}(t) v_x(t) dx \leq c \|v_x(t)\|_{L^2} \|v(t)\|_{L^2} \approx c \|v_0\|_{L^2} \|v\|_{L^6}^3.$$

So to obtain the small bound c , we turn to obtain the smallness of the quantity

$$f(t) := \|v(t)\|_{L^4}^4 / \|v(t)\|_{L^6}^3.$$

Indeed, we shall prove that $f(t)^2$ obeys some cubic inequality. Then we find that, the settlement to the global well-posedness is turned to find the solution to an elementary cubic equation.

2. THE PROOF OF THEOREM 1.1

Let v be the function in (1.8), which is the solution of the equation (1.9). Since

$$u_x = e^{i\frac{3}{4} \int_{-\infty}^x |v(t,y)|^2 dy} \left(i\frac{3}{4} |v|^2 v + v_x \right).$$

Therefore, by the sharp Gagliardo-Nirenberg inequality (see [17]),

$$\|f\|_{L^6}^6 \leq \frac{4}{\pi^2} \|f\|_{L^2}^4 \|f_x\|_{L^2}^2, \quad (2.1)$$

(which the equality is attained by Q), and mass conservation law, for any $t \in \mathbb{R}$,

$$\begin{aligned} \|u_x(t)\|_{L^2} &\leq \|v_x(t)\|_{L^2} + \frac{3}{4}\|v(t)\|_{L^6}^3 \leq \|v_x(t)\|_{L^2} + \frac{3}{2\pi}\|v(t)\|_{L^2}^2\|v_x(t)\|_{L^2} \\ &\leq \left(1 + \frac{3}{2\pi}\|u_0\|_{L^2}^2\right)\|v_x(t)\|_{L^2}. \end{aligned}$$

That is, the boundedness of u in H^1 -norm is equivalent to the boundedness of v in H^1 -norm. Therefore, to prove the theorem, we may consider the function v in (1.8) instead. To simply the notations, from now on, we set

$$E_0 = E(v_0), \quad P_0 = P(v_0), \quad m_0 = M_D(v_0).$$

Furthermore, we assume that $m_0 > 2\pi$. Otherwise, it has been proved the global well-posedness in [9, 18].

Let $(-T_-(v_0), T_+(v_0))$ be the maximal lifespan of the solution v of (1.9). To prove Theorem 1.1, it is sufficient to obtain the (indeed uniformly) *a priori* estimate of the solutions on H^1 -norm, that is,

$$\sup_{t \in (-T_-(v_0), T_+(v_0))} \|v_x(t)\|_{L^2} < +\infty.$$

As [18], we argue by contradiction. Suppose that there exists a sequence $\{t_n\}$ with $t_n \rightarrow -T_-(v_0)$, or $T_+(v_0)$, such that

$$\|v_x(t_n)\|_{L^2} \rightarrow +\infty, \text{ as } n \rightarrow \infty. \quad (2.2)$$

Now we define the sequence $\{f_n\}$ by

$$f_n = \frac{\|v(t_n)\|_{L^4}^4}{\|v(t_n)\|_{L^6}^3},$$

then $\|v(t_n)\|_{L^4}^4 = f_n\|v(t_n)\|_{L^6}^3$, and by the energy conservation law (1.10),

$$\|v(t_n)\|_{L^4}^8 = 16f_n^2(\|v_x(t_n)\|_{L^2}^2 - E_0). \quad (2.3)$$

Moreover, we give the lower and upper bounds of f_n . We denote C_{GN} to be the sharp constant of the following Gagliardo-Nirenberg inequality,

$$\|f\|_{L^6} \leq C_{GN}\|f\|_{L^4}^{\frac{8}{9}}\|f_x\|_{L^2}^{\frac{1}{9}}. \quad (2.4)$$

The best constant and the optimal functions for this inequality was obtained by Agueh [1]. More precisely, an optimal function is written as

$$\Psi(x) = \left(x^2 + 1\right)^{-\frac{1}{2}},$$

and the best constant $C_{GN} = 3^{\frac{1}{6}}(2\pi)^{-\frac{1}{9}}$. In particular, $\sqrt{2}\Psi$ is the unique (up to symmetries) radial ground state of the following elliptic equation,

$$\partial_{xx}\psi - \psi^3 + \frac{3}{4}\psi^5 = 0.$$

Now we have

Lemma 2.1. *There exists an $\varepsilon_n : \varepsilon_n \rightarrow 0$ as $n \rightarrow \infty$, such that*

$$2C_{GN}^{-\frac{9}{2}} + \varepsilon_n \leq f_n \leq \sqrt{m_0}. \quad (2.5)$$

Proof of Lemma 2.1. First, by Hölder's inequality, we have

$$\|v(t_n)\|_{L^4}^4 \leq \|v(t_n)\|_{L^2} \|v(t_n)\|_{L^6}^3 = \sqrt{m_0} \|v(t_n)\|_{L^6}^3,$$

and thus

$$f_n \leq \sqrt{m_0}.$$

Furthermore, by the sharp Gagliardo-Nirenberg (2.4) and the energy conservation law (1.10), we have

$$\begin{aligned} f_n &\geq \frac{\left(C_{GN}^{-6} \|v(t_n)\|_{L^6}^6 \|v_x(t_n)\|_{L^2}^{-\frac{2}{3}}\right)^{\frac{3}{4}}}{\|v(t_n)\|_{L^6}^3} \\ &= C_{GN}^{-\frac{9}{2}} \frac{\|v(t_n)\|_{L^6}^{\frac{3}{2}}}{\|v_x(t_n)\|_{L^2}^{\frac{1}{2}}} = 2C_{GN}^{-\frac{9}{2}} \frac{\|v(t_n)\|_{L^6}^{\frac{3}{2}}}{\left(\|v(t_n)\|_{L^6}^6 + 16E_0\right)^{\frac{1}{4}}} \\ &= 2C_{GN}^{-\frac{9}{2}} + \varepsilon_n, \end{aligned}$$

where

$$\varepsilon_n := 2C_{GN}^{-\frac{9}{2}} \|v(t_n)\|_{L^6}^{\frac{3}{2}} \frac{\|v(t_n)\|_{L^6}^{\frac{3}{2}} - \left(\|v(t_n)\|_{L^6}^6 + 16E_0\right)^{\frac{1}{4}}}{\|v(t_n)\|_{L^6}^{\frac{3}{2}} \left(\|v(t_n)\|_{L^6}^6 + 16E_0\right)^{\frac{1}{4}}}.$$

Note that $\varepsilon_n = O(\|v_n\|_{L^6}^{-6}) \rightarrow 0$, by the mean value theorem. \square

In spirit of the paper [2], we define

$$\phi(t, x) = e^{i\alpha x} v(t, x),$$

where the parameter α will be given later. Then $\phi_x(t, x) = e^{i\alpha x} (i\alpha v(t, x) + v_x(t, x))$, and thus

$$\|\phi_x\|_{L^2}^2 = \|v_x\|_{L^2}^2 + 2\alpha \operatorname{Im} \int \bar{v} v_x dx + \alpha^2 \|v\|_{L^2}^2.$$

Minus $\frac{1}{16} \|\phi\|_{L^6}^6 = \frac{1}{16} \|v\|_{L^6}^6$ in the two sides, yields that

$$E(\phi) = E(v) + 2\alpha \operatorname{Im} \int \bar{v} v_x dx + \alpha^2 \|v\|_{L^2}^2.$$

By mass, energy conservation laws (1.2) and (1.10), this gives that

$$-2\alpha \operatorname{Im} \int \bar{v} v_x dx = -E(\phi) + \alpha^2 m_0 + E_0. \quad (2.6)$$

On the other hand, by using (2.4), we have

$$\begin{aligned} E(\phi(t_n)) &= \|\phi_x(t_n)\|_{L^2}^2 - \frac{1}{16} \|\phi(t_n)\|_{L^6}^6 \\ &\geq C_{GN}^{-18} \|\phi(t_n)\|_{L^6}^{18} \|\phi(t_n)\|_{L^4}^{-16} - \frac{1}{16} \|\phi(t_n)\|_{L^6}^6 \\ &= \left(C_{GN}^{-18} \|v(t_n)\|_{L^6}^{12} \|v(t_n)\|_{L^4}^{-16} - \frac{1}{16}\right) \|\phi(t_n)\|_{L^6}^6 \\ &= \left(C_{GN}^{-18} f_n^{-4} - \frac{1}{16}\right) \|v(t_n)\|_{L^6}^6. \end{aligned}$$

Therefore, this combining with (2.6), gives that

$$-2\alpha \operatorname{Im} \int \bar{v}(t_n, x) v_x(t_n, x) dx \leq \left(\frac{1}{16} - C_{GN}^{-18} f_n^{-4} \right) \|v(t_n)\|_{L^6}^6 + \alpha^2 m_0 + E_0.$$

This implies that for $\alpha > 0$,

$$-\operatorname{Im} \int \bar{v}(t_n, x) v_x(t_n, x) dx \leq \left(\frac{1}{16} - C_{GN}^{-18} f_n^{-4} \right) \|v(t_n)\|_{L^6}^6 \cdot \frac{1}{2\alpha} + \frac{1}{2} \alpha m_0 + \frac{1}{2} \alpha^{-1} E_0. \quad (2.7)$$

We consider the case of $\frac{1}{16} - C_{GN}^{-18} f_n^{-4} < 0$ first. Then by the momentum conservation law (1.11), we have

$$\frac{1}{4} \|v(t_n)\|_{L^4}^4 = -\operatorname{Im} \int \bar{v}(t_n, x) v_x(t_n, x) dx + P_0. \quad (2.8)$$

Hence by (2.7) and choosing $\alpha = 1$, we obtain

$$\|v(t_n)\|_{L^4}^4 \leq 2(m_0 + E_0 + 2P_0).$$

Therefore, by (2.3) and Lemma 2.1, we have the boundedness of $\|v_x(t_n)\|_{L^2}$. This is a contradiction with (2.2).

Now we consider the case of $\frac{1}{16} - C_{GN}^{-18} f_n^{-4} \geq 0$. We set

$$\alpha = \frac{1}{4} \sqrt{m_0^{-1} \left(1 - 16C_{GN}^{-18} f_n^{-4} \right)} \|v(t_n)\|_{L^6}^3,$$

then $\alpha = \alpha_n \rightarrow \infty$ as $n \rightarrow \infty$, and (2.7) turns to

$$-\operatorname{Im} \int \bar{v}(t_n, x) v_x(t_n, x) dx \leq \frac{1}{4} \sqrt{m_0 \left(1 - 16C_{GN}^{-18} f_n^{-4} \right)} \|v(t_n)\|_{L^6}^3 + \frac{1}{2} \alpha_n^{-1} E_0. \quad (2.9)$$

Note that the remainder term

$$\frac{1}{2} \alpha_n^{-1} E_0 \rightarrow 0, \quad \text{as } n \rightarrow \infty.$$

By (2.8) and (2.9),

$$\|v(t_n)\|_{L^4}^4 \leq \sqrt{m_0 \left(1 - 16C_{GN}^{-18} f_n^{-4} \right)} \|v(t_n)\|_{L^6}^3 + 2\alpha_n^{-1} E_0 + 4P_0.$$

This implies that

$$f_n \leq \sqrt{m_0 \left(1 - 16C_{GN}^{-18} f_n^{-4} \right)} + (2\alpha_n^{-1} E_0 + 4P_0) \|v(t_n)\|_{L^6}^{-3}.$$

Furthermore, it gives that

$$f_n^6 \leq m_0 f_n^4 - 16m_0 C_{GN}^{-18} + f_n^4 \mathcal{R}_n, \quad (2.10)$$

where

$$\mathcal{R}_n = 2 \sqrt{m_0 \left(1 - 16C_{GN}^{-18} f_n^{-4} \right)} (2\alpha_n^{-1} E_0 + 4P_0) \|v(t_n)\|_{L^6}^{-3} + (2\alpha_n^{-1} E_0 + 4P_0)^2 \|v(t_n)\|_{L^6}^{-6}.$$

By the lower and upper boundedness of f_n from Lemma 2.1, we have

$$f_n^4 \mathcal{R}_n = O(\|v(t_n)\|_{L^6}^{-3}) \rightarrow 0, \quad \text{as } n \rightarrow \infty.$$

Thus for any small and fixed $\epsilon > 0$, by choosing n large enough, we have $f_n^4 \mathcal{R}_n < \epsilon$. Hence (2.10) becomes

$$f_n^6 \leq m_0 f_n^4 - 16m_0 C_{GN}^{-18} + \epsilon. \quad (2.11)$$

Let $X = f_n^2$, then (2.11) turns to the inequality

$$X^3 - m_0 X^2 + b < 0, \quad (2.12)$$

where $b = 16m_0 C_{GN}^{-18} - \epsilon > 0$. Let

$$F(X) = X^3 - m_0 X^2 + b,$$

then the function $F(X)$ attains its minimum value at $\frac{2}{3}m_0$ in the region of $[0, \infty)$. Therefore, there are two positive solutions X_1 and X_2 solve the equation

$$X^3 - m_0 X^2 + b = 0. \quad (2.13)$$

if and only if $F(\frac{2}{3}m_0) < 0$. In another word, the inequality (2.12) has no solution in the region of $[0, +\infty)$ if and only if

$$F(\frac{2}{3}m_0) \geq 0. \quad (2.14)$$

Now the condition (2.14) is equivalent to

$$\frac{8}{27}m_0^3 - \frac{4}{9}m_0^3 + b \geq 0.$$

By the arbitrariness of ϵ , it is deduced to $m_0 < 6\sqrt{3}C_{GN}^{-9} = 4\pi$. Therefore, we obtain that the problem (1.9) is global well-posedness when $m_0 < 4\pi$. This proves our main theorem.

One may expect to get some profit from the restriction $X \in (4C_{GN}^{-9}, m_0)$ (rather than $[0, +\infty)$), according to Lemma 2.1. However, we will explain below that we in fact can not get any more from this. To see this, we note that in this case of $m_0 \geq 4\pi$, (2.12) is solved in the region of $[0, +\infty)$ by

$$X_1 < X < X_2.$$

Now we claim that

$$4C_{GN}^{-9} < X_1 < X_2 < m_0. \quad (2.15)$$

Indeed, first we observe that when $m_0 \geq 4\pi$,

$$\frac{2}{3}m_0 \geq \frac{8}{3}\pi > 4C_{GN}^{-9} = \frac{8}{3\sqrt{3}}\pi.$$

Moreover, by choosing ϵ small, we have

$$F(4C_{GN}^{-9}) = 64C_{GN}^{-27} - \epsilon > 0.$$

These two facts imply that $4C_{GN}^{-9} < X_1$. Similarly, since

$$m_0 < \frac{2}{3}m_0; \quad \text{and} \quad F(m_0) = b > 0,$$

we have $X_2 < m_0$. In conclusion, we have the claim (2.15). Therefore, the inequality (2.12) is always solvable in the region of $(4C_{GN}^{-9}, m_0)$ when $m_0 \geq 4\pi$. So we can not get the contradiction from the restriction of $(4C_{GN}^{-9}, m_0)$.

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